MATHEMATICS APPLICATIONS UNITS 3&4
Section One:
Calculator-free

SOLUTIONS

WA student number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1.　　The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.　　Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.　　You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.　　Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.　　It is recommended that you do not use pencil, except in diagrams.

6.　　Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.　　The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

A sample of fifteen people were asked to take between 30 and 90 minutes to practice a new skill and then their percentage error score in performing the skill E recorded against their practice time T minutes. The results are shown on the scatterplot below.
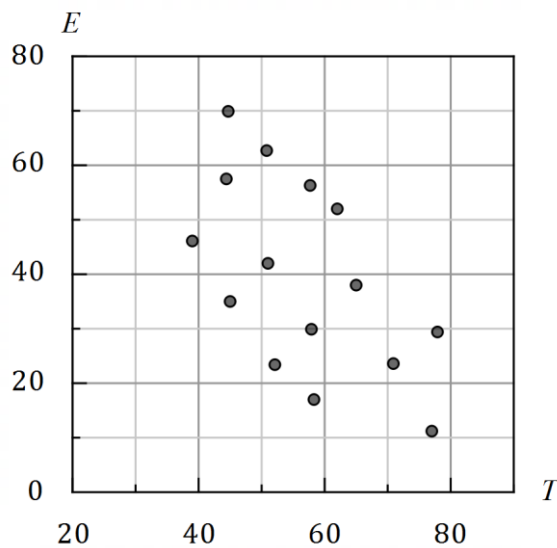
(a) Describe the association between T and E in terms of direction, form and strength.

(3 marks)

Solution
Direction is **negative**, form is **linear**, and strength is **moderate**.

Specific behaviours
✓ direction
✓ form
✓ strength

(b) (i) Estimate, to one decimal place, the value of the correlation coefficient between the variables. (1 mark)

Solution
$r \approx -0.6 \Rightarrow r^2 = 0.36$
Specific behaviours
✓ estimate to 1dp between -0.4 and -0.8 inclusive

(ii) Hence or otherwise, determine the percentage of the variation in the error scores that can be explained by the variation in the practice times for this sample.
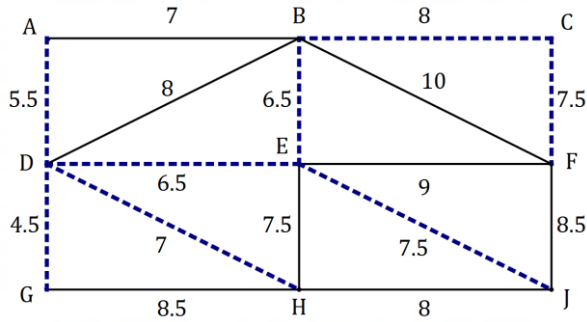
(1 mark)

Solution
36% of the variation in E can be explained by the variation in T.

Specific behaviours
✓ correctly squares r and writes as percentage

See next page

Question 2   (7 marks)

The vertices on the graph below represent nine pumping stations. The edge weights are the times required to install new electrical cabling between connected stations.

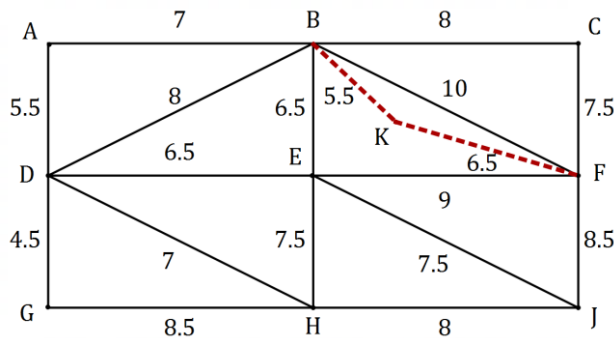| Solution (a) |
|---|
| See graph |
| **Specific behaviours** |
| ✓ any spanning tree |
| ✓ minimum spanning tree |

(a)   Clearly show the minimum spanning tree on the graph.   (2 marks)

A contractor charges $200 per hour to install the cabling.

(b)   Determine the cost to install new electrical cabling using the minimum spanning tree.

(2 marks)

Solution
$L = 4.5 + 5.5 + 6.5 + 6.5 + 7 + 7.5 + 8 + 7.5$
$\quad = 53 \text{ h}$
$\qquad\qquad C = 200 \times 53$
$\qquad\qquad\quad = \$10\,600$
Specific behaviours
✓ length of MST in hours
✓ correct cost

(c)   A tenth pumping station K is to be included. The time to install cable between K and B is 5.5 hours and between K and F is 6.5 hours. *Note: a second copy of the diagram has been provided to use while answering question (c).*

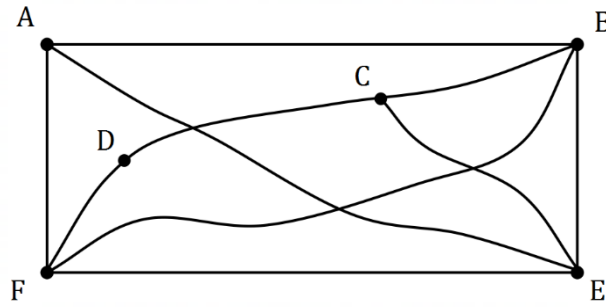| Solution (c)(i) |
|---|
| See graph |
| **Specific behaviours** |
| ✓ correctly adds edges, weights |

(i)   Use this information to add pumping station K to the second copy of the graph provided above.   (1 mark)

(ii)   If the new cabling is now installed using the minimum spanning tree that includes K, determine the extra cost of the installation.   (2 marks)

Solution
Changes to minimum spanning tree: add BK and KF, drop BC.
Change to length of minimum spanning tree is $5.5 + 6.5 - 8 = 4$.
Hence extra cost is $4 \times 200 = \$800$.
Specific behaviours
✓ indicates changes to minimum spanning tree
✓ correct extra cost

See next page

Question 3   **(7 marks)**

Graph G is shown at right.

(a)   Adding all missing vertices to those shown below, draw graph G in the plane, to clearly show that it is planar   (2 marks)

Solution
  (Example only)
Specific behaviours
✓ no edges that cross, at least 4 correct vertex degrees
✓ correctly drawn in the plane

(b)   Show that graph G satisfies Euler's formula.   (2 marks)

Solution
$v = 6, e = 10, f = 6$ $v + f - e = 6 + 6 - 10 = 2$
Specific behaviours
✓ correctly counts v, e, f
✓ substitutes into Euler's formula and simplifies

(c)   Graph G is semi-Eulerian. Describe two features of the trail it contains to be classified as semi-Eulerian.   (2 marks)

Solution
The trail is open and includes every edge exactly once.   *(Only accept 'trail starts and ends at odd degree vertices' instead of 'trail is open' if answer makes clear the odd vertices are different.)*
Specific behaviours
✓ indicates trail is open
✓ indicates trail includes every edge exactly once

(d)   Describe where an edge can be added to graph G so that it contains an Eulerian trail.

                                                (1 mark)

Solution
Add edge between vertex A and vertex C.
Specific behaviours
✓ indicates correct vertices

Question 4   **(6 marks)**

Arif, Brenton, Carter and Dana have been chosen for the 4×50 m medley relay team in a swimming carnival. This relay is swum by four different swimmers, each swimming one of four different strokes. Their best times, in seconds, to swim 50 m freestyle, backstroke, breaststroke and butterfly are shown in the following table.

	Freestyle	Backstroke	Breaststroke	Butterfly
Arif	32	42	36	35
Brenton	33	43	38	37
Carter	31	42	36	35
Dana	32	41	35	36

(a) Show use of the Hungarian algorithm to determine which stroke each boy should swim so that the team completes the 4×50 m medley relay in the shortest possible time.

(4 marks)

Solution
$$\begin{bmatrix} 0 & 10 & 4 & 3 \\ 0 & 10 & 5 & 4 \\ 0 & 11 & 5 & 4 \\ 0 & 9 & 3 & 4 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 1 & \mathbf{0} \\ 0 & \mathbf{0} & 1 & 0 \\ \mathbf{0} & 1 & 1 & 0 \\ 1 & 0 & \mathbf{0} & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 0 & 0 & \mathbf{0} \\ 0 & \mathbf{0} & 1 & 1 \\ \mathbf{0} & 1 & 1 & 1 \\ 1 & 0 & \mathbf{0} & 2 \end{bmatrix}$$
Assignment: Arif - Butterfly; Brenton - Backstroke; Carter - Freestyle; Dana - Breastroke.
Specific behaviours
✓ reduces rows
✓ reduces columns
✓ reduces again so that four lines needed to cover
✓ states assignment

(b) The record for the 4×50 m medley relay is 2 minutes and 23 seconds. If all the boys swim their assigned leg in their best time, will they break the record? Justify your answer.
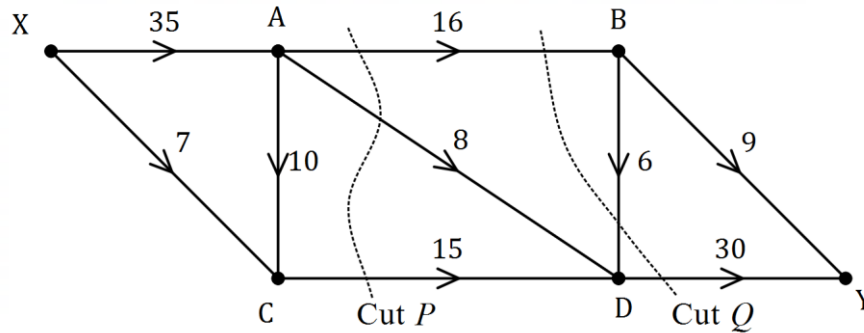
(2 marks)

Solution
No. Their time will be $35 + 43 + 31 + 35 = 144$ seconds but the record is 143 seconds, which is 1 second quicker.
Specific behaviours
✓ indicates time for boys
✓ compares to record and states no

Question 5 (8 marks)

A directed subgraph of a distribution network is shown below. The vertices represent distribution centres and the weight on each edge is the maximum volume of parcels, in cubic metres, that can be transported from one distribution centre to another every day.

(a) Determine the capacity of cut P and the capacity of cut Q shown above. (2 marks)

Solution
Cut P is $16 + 8 + 15 = 39$ m^3.
Cut Q: $16 + (-6) + 30 = 46$ m^3.

Specific behaviours
✓ cut P; ✓ cut Q

(b) Determine the maximum volume of parcels that can be transported

(i) from centre X to centre C in a day. (1 mark)

Solution
Maximum volume is 17 m^3.

Specific behaviours
✓ correct maximum flow

(ii) from centre A to centre D in 5 days. (2 marks)

Solution
Minimum cut is 24 and so maximum volume in 5 days is $24 \times 5 = 120$ m^3.

Specific behaviours
✓ minimum cut; ✓ maximum volume

(c) Determine the maximum volume of parcels that can be transported from centre X to centre Y in 10 days. (3 marks)

Solution	Alternative Solution
Cut $BY, DY = 39$, cut $XA, XC = 42$, cut $BY, BD, AD, CD = 38$ Minimum cut is 38. Maximum volume in 10 days is $10 \times 38 = 380$ m^3 of parcels.	$XABY = 9$, $XABDY = 6$, $XADY = 8$, $XACDY = 10$, $XCDY = 5$. Maximum flow is 38. Maximum volume in 10 days is $10 \times 38 = 380$ m^3 of parcels.

Specific behaviours	Specific behaviours
✓ at least two more cuts ✓ indicates minimum cut ✓ correct volume of parcels	✓ systematically lists flows ✓ correct maximum flow ✓ correct volume of parcels

Question 6   **(6 marks)**

Lucas the Lupin farmer is experimenting with a new species of Lupin plant. He varied the amount of fertiliser (F), in grams used during each week and recorded the number of kilograms (L) of Lupin yielded per 100 square meters. Lucas was tasked with writing a report to present to other local farmers explaining the use of his linear model that was fitted to the data set.

Lucas' report included the following summary.

$$r_{FL} = 0.78 \text{ and } \hat{L} = 13F + 150$$

(a)   Identify the explanatory variable.   (1 mark)

Solution
The amount of fertiliser used is the explanatory variable

Specific behaviours
✓ correct variable

(b)   Use the equation of the least-squares regression line to predict the total number of kilograms of Lupin produced when 10 grams of fertiliser is used.   (2 marks)

Solution
$\hat{L} = 13(10) + 150$ $= 280$ kilograms per 100 square metres

Specific behaviours
✓ subs 10 into equation   ✓ states 280

Plot A   Plot B Plot C
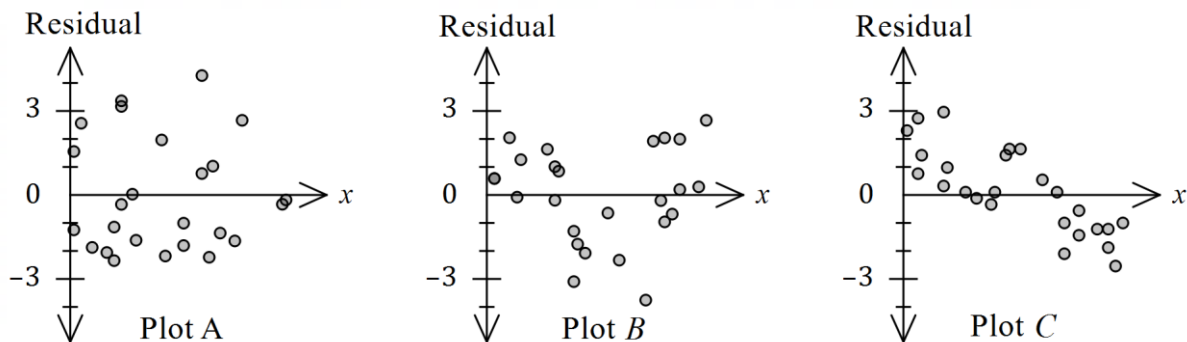
When Lucas started writing a report of his findings, he realised that he had accidently included three residual plots in his report. One of them is the correct plot for his linear regression and the other two are residual plots for other data sets where linear regression is not appropriate.

(c)   State, with justification, which residual plot is most likely to be the correct plot for Lucas' report. Explain why the other two plots are not likely to be the correct plots.   (3 marks)
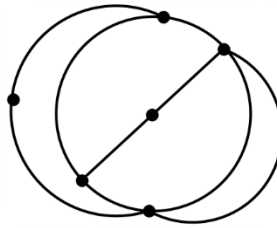
Solution
Plot A is most likely to be the correct plot for Lucas' linear model as there appears to be no pattern in the residual plot. Plots B and C both appear to have patterns in the residuals.

Specific behaviours
✓ Plot A is most likely to be correct   ✓ justification for A   ✓ explains the other two plots contain patterns

See next page

Question 7 **(6 marks)**

Graph G is shown.

It represents a communication network of a group of six diplomats.

(a) State, with reasons, if graph G is a simple graph. (2 marks)

Solution
No - it contains multiple edges.
Specific behaviours
✓ states no
✓ explanation

(b) For graph G, determine the length of the longest

(i) closed trail it contains. (1 mark)

Solution
Length is 8.
Specific behaviours
✓ correct length

(ii) open path it contains. (1 mark)

Solution
Length is 5.
Specific behaviours
✓ correct length

(c) Explain why graph G is a Hamiltonian graph. (2 marks)

Solution
Graph G contains a cycle (closed path) that includes each vertex in the graph.
Specific behaviours
✓ indicates closed path
✓ indicates path includes each vertex

Question 8   **(7 marks)**

A project involves the completion of activities A to H, as shown in the following table. Note that only three of the activity durations are shown.

Activity	A	B	C	D	E	F	G	H
Immediate predecessor(s)	–	–	A	A	B	B	D, E	E, F
Duration (weeks)		9	24				13	
Float (weeks)	2	0	11	2	0	9	0	4

The minimum completion time for the project is 38 weeks.

(a)   Construct an activity network to represent the above information.   (3 marks)

Solution
Specific behaviours
✓ shows most activities as labelled edges
✓ correct predecessors for each activity, including dummy edge
✓ correct activity network, with direction indicated

(c)   List the tasks that lie on the critical path.   (1 mark)

Solution
The tasks are B, E and G (zero float).
Specific behaviours
✓ correct tasks

(b)   Determine a possible duration for each of the activities A, D, E, F and H.   (3 marks)

Solution
Using critical path: $e = 38 - 13 - 9 = 16$.
Using Duration=LST of next−EST−Float:
$$a = EST_C = 38 - 24 - 11 = 3, \qquad d = (38 - 13) - 3 - 2 = 20$$
$$h = 38 - (38 - 13) - 4 = 9, \qquad f = (38 - 9) - 9 - 9 = 11$$
Hence durations of A, D, E, F and H are $3, 20, 16, 11$ and 9 weeks respectively.
Specific behaviours
✓ duration of E
✓ durations of A and D
✓ durations of H and F

End of questions

Supplementary page

Question number: _____